

FALL SPRING 2017

$$1. \int_C \frac{x+2y}{\sqrt{9x^4+1}} ds$$

$$y=x^3, 0 \leq x \leq 1$$

$$\int_0^1 \frac{x+2t^3}{9t^4+1} dt$$

$$= \int_0^1 \frac{t}{\sqrt{9t^4+1}} dt + \int_0^1 \frac{2t^3}{\sqrt{9t^4+1}} dt$$

$$\int_0^1 \frac{t}{\sqrt{9t^4+1}} dt$$

$$\text{Let } u=t^2 \quad \frac{du}{2} = t dt$$

$$= \frac{1}{2} \int_0^1 \frac{1}{\sqrt{9u^2+1}} du$$

$$u = \frac{1}{3} \tan(v) \quad du = \frac{1}{3} dv$$

$$= \frac{1}{2} \int_0^1 \frac{\sec(v)}{3} dv = \frac{1}{6} \int \sec(v) dv$$

$$= \frac{1}{6} \ln |\tan(v) + \sec(v)| dv$$

$$= \frac{1}{6} \ln |\tan(\tan^{-1}(3t^2)) + \sec(\tan^{-1}(3t^2))| \Big|_0^1$$

$$= \frac{1}{6} \ln |3t^2 + \sqrt{1+9t^4}| \Big|_0^1$$

$$= \int_0^1 \frac{2t^3}{\sqrt{9t^4+1}} dt$$

$$\text{Let } t^4 = u \\ du = 4t^3 dt \\ \frac{du}{4} = t^3 dt$$

$$= \frac{2}{4} \int_0^1 \frac{1}{\sqrt{9u+1}} du$$

$$v = 9u+1 \quad dv = 9 du$$

$$= \frac{1}{2} \int_0^1 \frac{1}{9\sqrt{v}} dv = \frac{1}{2} \times \frac{1}{9} \left(\frac{v^{\frac{1}{2}}}{\frac{1}{2}} \right)$$

$$= \frac{1}{9} \sqrt{9t^4+1} \Big|_0^1$$

$$= \frac{1}{6} \ln |3t^2 + \sqrt{1+9t^4}| + \frac{1}{9} \sqrt{9t^4+1} \Big|_0^1$$

$$= \frac{\ln(3+\sqrt{10})}{6} + \frac{\sqrt{10}-1}{6}$$

2. $\int_C F \cdot dr$ $F(x,y) = \left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle$

$$y = r \sin \theta \quad x = r \cos \theta, \quad x^2 + y^2 = r^2$$

$$dy = r \cos \theta d\theta \quad dx = -r \sin \theta d\theta$$

$$= \int_0^{2\pi} \frac{-r \sin \theta \cdot (-r \sin \theta d\theta) + \frac{r \cos \theta}{r^2} (r \cos \theta d\theta)}{r^2}$$

$$= \int_0^{2\pi} \sin^2 \theta + \cos^2 \theta d\theta = \int_0^{2\pi} 1 d\theta = \theta \Big|_0^{2\pi}$$

$$= 2\pi$$

$$3 \int_C 2y dx + x dy \quad (0,0) \text{ to } (1,5)$$

$$L: (0,0) + t(1,5) = (x,y) \\ = (t, 5t)$$

$$x = t \quad y = 5t$$

$$dx = dt \quad dy = 5dt$$

$$\int 2(5t)dt + t(5dt)$$

$$= \int 10t dt + 5t dt$$

$$= \int_0^1 15t dt = \left. \frac{15t^2}{2} \right|_0^1$$

$$= \frac{15}{2}$$

$$4 \int_C y dx + x dy \quad (0,0) \quad (4,5)$$

$$L: (0,0) + t(4,5)$$

$$(4t, 5t)$$

$$x = 4t \\ dx = 4dt$$

$$y = 5t \\ dy = 5dt$$

$$\int_0^4 5t \cdot 4dt + 4t \cdot 5dt$$

$$\int_0^4 20t dt + 20t dt = \int_0^4 40t dt$$

$$= 20t^2 \Big|_0^4$$

$$= 20(4)^2$$

$$= 320$$

$$4 \int_C y dx + x dy$$

(0,0) to (4,5)

$$5 \quad x = 4 \cos t \quad y = 4 \sin t \quad -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$$

$$dx = -4 \sin t dt \quad dy = 4 \cos t dt$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 4 \sin t (-4 \sin t dt) + (4 \cos t) 4 \cos t dt$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} -16 \sin^2 t + 16 \cos^2 t$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 16 \cos^2 t - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 16 \left(\frac{1 - 2 \cos(2t)}{2} \right) dt$$

$$\int 16 \cos^2 t \, dt = \frac{16}{2} \int (1 + \cos(2t)) \, dt$$

$$= 8t + \frac{8}{2} \sin 2t$$

$$\int 16 \left(\frac{1 - \cos 2t}{2} \right) dt = 8 \left(t - \frac{1}{2} \sin(2t) \right)$$

$$= 8 + \frac{8}{2} \sin 2t + 8t + \frac{8}{2} \sin 2t$$

$$= 8 \sin 2t \Big|_{-\pi/2}^{\pi/2}$$

$$= 0$$

$$G \quad r(u, v) = \langle u^2, uv, \frac{1}{2}v^2 \rangle \quad 0 \leq u \leq 1, 0 \leq v \leq 2$$

$$r_u = \left\langle \frac{\partial(u^2)}{\partial u}, \frac{\partial(uv)}{\partial u}, \frac{\partial(\frac{1}{2}v^2)}{\partial u} \right\rangle = \langle 2u, v, 0 \rangle$$

$$r_v = \left\langle \frac{\partial(u^2)}{\partial v}, \frac{\partial(uv)}{\partial v}, \frac{\partial(\frac{1}{2}v^2)}{\partial v} \right\rangle = \langle 0, u, v \rangle$$

$$\begin{vmatrix} i & j & k \\ 2u & v & 0 \\ 0 & u & v \end{vmatrix} = v^2 - 2uv + 2u^2$$

$$\int_0^1 \int_0^2 (v^2 - 2uv + 2u^2) \, dv \, du.$$

$$\int_0^2 \left. \frac{v^3}{3} - uv^2 + 2u^2v \right|_0^2$$

$$\int_0^1 \left(\frac{8}{3} + 4u^2 - 4u \right) du$$

$$= \left. \frac{8}{3}u + \frac{4}{3}u^3 - 2u^2 \right|_0^1$$

$$= 2$$

$$7 \quad z = \sqrt{36 - x^2 - y^2}$$

$$x^2 + y^2 + z^2 = 36$$

$$r(\phi, \theta) = \langle 6 \sin \phi \cos \theta, 6 \sin \phi \sin \theta, 6 \cos \phi \rangle$$

$$r_\phi = \langle 6 \cos \phi \cos \theta, 6 \cos \phi \sin \theta, -6 \sin \phi \rangle$$

$$r_\theta = \langle -6 \sin \phi \sin \theta, 6 \sin \phi \cos \theta, 0 \rangle$$

$$r_\phi \times r_\theta = \langle 36 \sin^2 \phi \cos \theta, 36 \sin^2 \phi \sin \theta, 0 \rangle$$

$$|r_\phi \times r_\theta| = 36 \sin \theta$$

$$\int_0^{\pi/2} \int_0^{2\pi} 6 \cos \phi (36 \sin \phi) d\theta d\phi$$

$$= \int_0^{\pi/2} \int_0^{2\pi} 216 \cos \phi \sin \phi d\theta d\phi$$

$$\int_0^{\pi/2} 216 \cos \phi \sin \phi \theta \bigg|_0^{2\pi}$$

$$\int_0^{\pi/2} 216 \pi \sin(2\phi) d\phi = -108 \pi \cos(\phi) \bigg|_0^{\pi/2}$$

$$= 216 \pi$$

$$8 \quad r(u, v) = \left\langle u, v, \frac{2}{u} \right\rangle$$

$$r_u = \left\langle 1, 0, -\frac{1}{u^2} \right\rangle$$

$$r_v = \left\langle 0, 1, 0 \right\rangle$$

$$|r_u \times r_v| = \begin{vmatrix} i & j & k \\ 1 & 0 & -\frac{1}{u^2} \\ 0 & 1 & 0 \end{vmatrix}$$

$$= \left\langle -\frac{1}{u^2}, 0, 1 \right\rangle$$

$$F(r(u, v)) = \langle v^7, u^7, 0 \rangle$$

$$= \int_1^2 \int_0^3 \frac{v^7}{u^2} dv du$$

$$= \int_1^2 \left. \frac{1}{8u^2} v^8 \right|_0^3 du = \int_1^2 \frac{6561}{8u^2} du$$

$$= -\frac{6561}{8u} \Big|_1^2 = -\frac{6561}{16} + \frac{6561}{8}$$

$$= \frac{6561}{16}$$

$$9. F(x, y, z) = \langle x+y+2z, 2x+y+z, x+2y+z \rangle$$

$$r(t) = \langle 1, 0, 0 \rangle + t(\langle 0, 1, 0 \rangle - \langle 0, 1, 0 \rangle)$$

$$r(t) = \langle 1-t, t, 0 \rangle$$

$$dr(t) = \langle -t, 1, 0 \rangle dt$$

$$\begin{aligned} F(r(t)) &= \langle 1-t+t, 2(1-t)+t, 1-t+2t \rangle \\ &= \langle 1, 2-t, 1+t \rangle \end{aligned}$$

$$\begin{aligned} F(r(t)) \cdot dr &= \langle 1, 2-t, 1+t \rangle \cdot \langle -t, 1, 0 \rangle dt \\ &= \langle -t+2-t \rangle dt \\ &= \langle -2t+2 \rangle dt \end{aligned}$$

$$\begin{aligned} &= \int_0^1 F \cdot dr = \int_0^1 (-2t+2) dt \\ &= -t^2 + 2t \Big|_0^1 \\ &= 1 \end{aligned}$$

$$10 \quad 0 \leq u \leq 3, \quad 0 \leq v \leq 2\pi$$

$$r(u, v) = \langle u \cos(v), u \sin(v), u \rangle$$

$$r_u = \langle \cos v, \sin v, 1 \rangle$$

$$r_v = \langle -u \sin(v), u \cos(v), 0 \rangle$$

$$|r_v \times r_u| = \begin{vmatrix} i & j & k \\ \cos v & \sin v & 1 \\ -u \sin v & u \cos v & 0 \end{vmatrix}$$

$$= -u \cos(v) - u \sin(v) + u \cos^2 v + u \sin^2 v$$

$$= \langle -u \cos(v) - u \sin(v) + u \rangle$$

$$= \int_0^3 \int_0^{2\pi} (-u \cos(v) - u \sin(v) + u) dv du$$

$$\int_0^3 u v = u \sin(v) + u \cos(v) \Big|_0^{2\pi}$$

$$= \int_0^3 2\pi u du = \pi u^2 \Big|_0^3$$

$$= 9\pi$$